Interpolating bitvector arithmetic constraints in MCSAT (preliminary report)

Stéphane Graham-Lengrand and Dejan Jovanović

SMT workshop, 7th July 2019

Outline

1. What is this about?
2. Why are we interested in this?
3. How do we do this?

1. What is this about?

A particular form of interpolation

Interpolation is usually considered between

- a formula $\mathcal{A}$
- a formula $\mathcal{B}$ such that $\mathcal{A}, \mathcal{B} \models \bot$,

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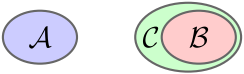

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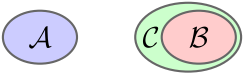

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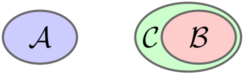

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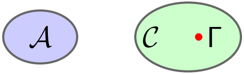

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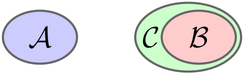

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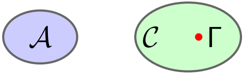

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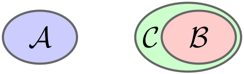

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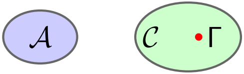

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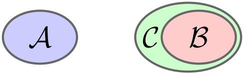

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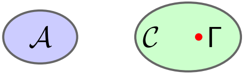

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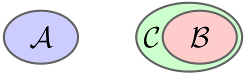

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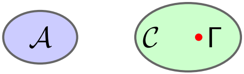

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Where do we find such problems?
… in model-constructing satisfiability (MCSAT)

2. Why are we interested in this?

The model-constructing approach to SMT-solving 1/2

MCSAT introduced in [dMJ13, JBdM13, Jov17],
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The template is a generalisation of how CDCL works.

Run = alternation of search phases and conflict analysis phases

Boolean theory can be given the same status as other theories.

The model-constructing approach to SMT-solving 2/2

- **Method**: as in CDCL for Boolean logic,
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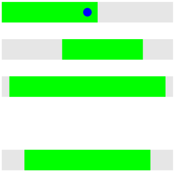

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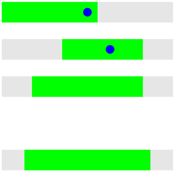

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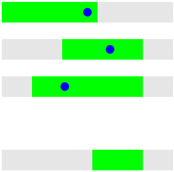

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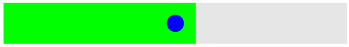
$\{x_1\}$	$C_1^1, \ldots, C_j^1, \ldots$		x_1
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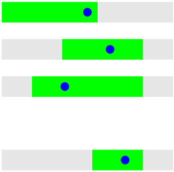
$\{x_1\}$	$C_1^1, \ldots, C_j^1, \ldots$		x_1
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SAT

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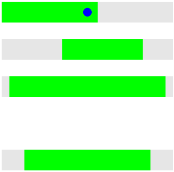

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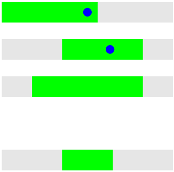

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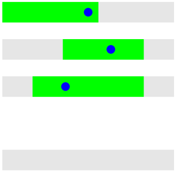

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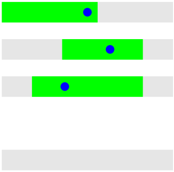

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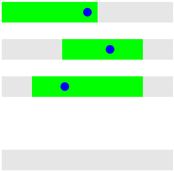

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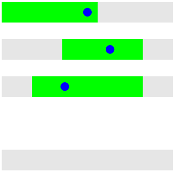
$\{x_1\}$	$C_1^1, \ldots, C_j^1, \ldots$		x_1
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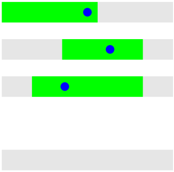

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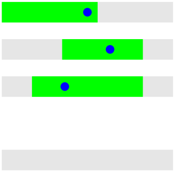

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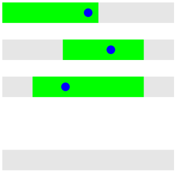

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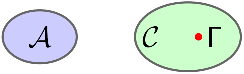
$\mathcal{A} \wedge \mathcal{C} \Rightarrow \bot$ (or equivalently quantif.-free $C_1 \wedge \cdots \wedge C_m \wedge \mathcal{C} \Rightarrow \bot$)



MCSAT theories

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- a nice way of representing domains of feasible values,
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Bitvectors in MCSAT first looked at in [ZWR16]

"*Interpolants do have applications in mcBV, e.g., for conflict generalization, but we do not currently employ such methods.*"

This is what we do now.

3. How do we do this?

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    - interpolants close to the bit level do not give understandable explanations of what was wrong about Γ; we'd rather have interpolants close to the word level

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Today: (a fragment of) linear bitvector arithmetic

Constraints (literals)	C	$::=$	$a \mid \neg a$
Atoms	a	$::=$	$t_1^w \leq^s t_2^w \mid t_1^w \leq^u t_2^w \mid t_1^w \simeq t_2^w$
Terms of bitwidth w	t^w	$::=$	$c_0^w + \Sigma_{i=1}^n c_i^w \cdot x_i^w$

Furthermore, when interpolating $\exists y (C_1 \wedge \cdots \wedge C_m)$, for each C among $C_1, \ldots, C_m$, coefficient c (resp. c') of y in t_1 (resp. t_2) are in $\{-1, 0, 1\}$ such that $c \cdot c' \in \{0, 1\}$.

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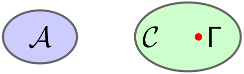

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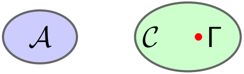
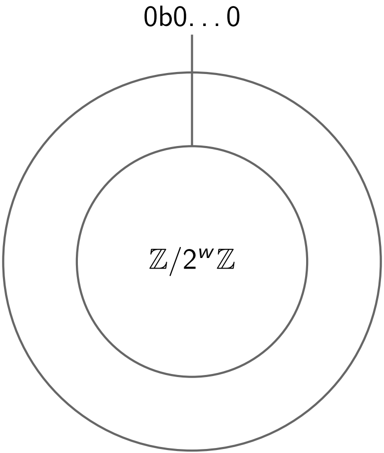
  - $E_1, \ldots, E_p \models$ "$(\bigcup_{i=1}^{n} I_i) = (\mathbb{Z}/2^w\mathbb{Z})$" i.e., $\mathcal{A}, E_1, \ldots, E_p \models \bot$

  We take $\mathcal{C}$ to be $E_1 \wedge \cdots \wedge E_p$



Our intervals are taken modulo 2^w (i.e., they may overflow):
$[0b1111; 0b0001[$ contains two values, namely $0b1111$ and $0b0000$

Example



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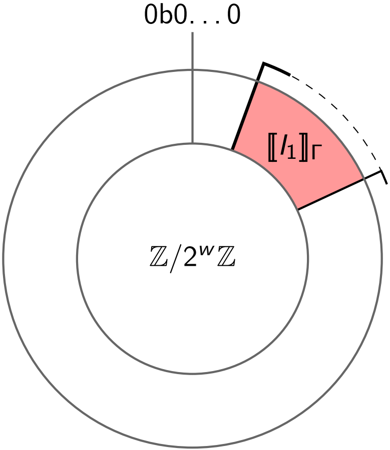

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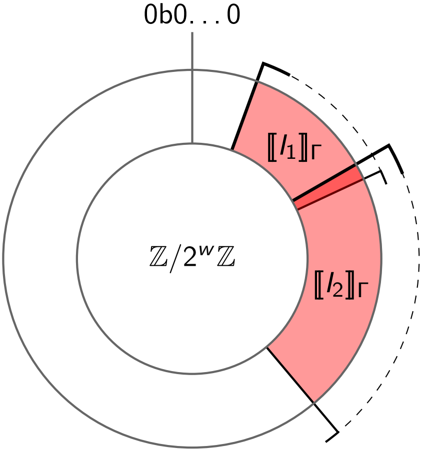

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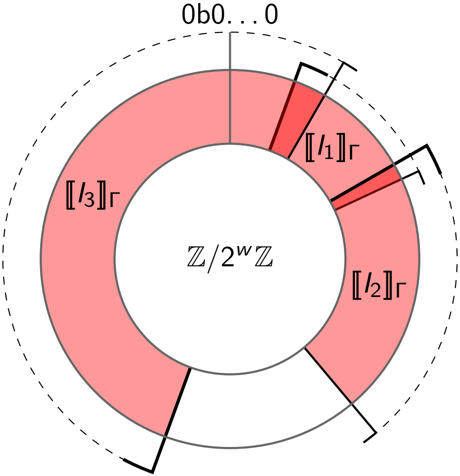

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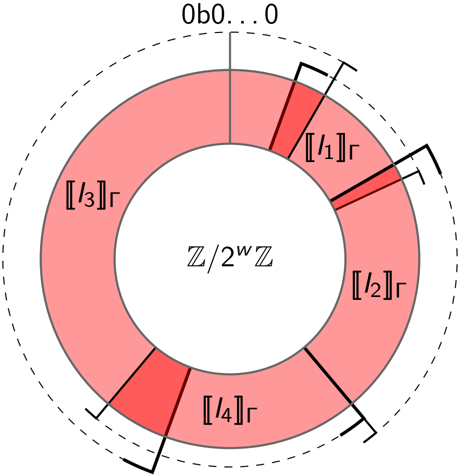

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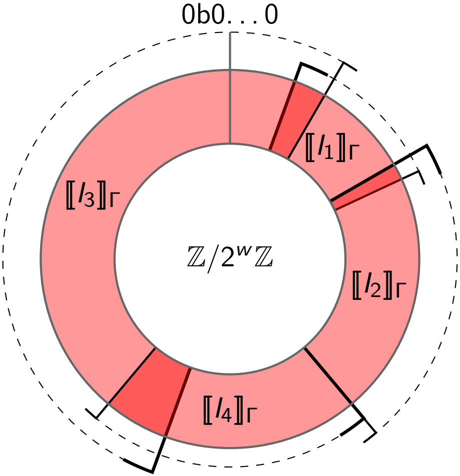

Coverage of $\mathbb{Z}/2^w\mathbb{Z}$ is full because:
$[\![u_1]\!]_\Gamma \in [\![l_2]\!]_\Gamma$ and $[\![u_2]\!]_\Gamma \in [\![l_4]\!]_\Gamma$ and $[\![u_4]\!]_\Gamma \in [\![l_3]\!]_\Gamma$ and $[\![u_3]\!]_\Gamma \in [\![l_1]\!]_\Gamma$

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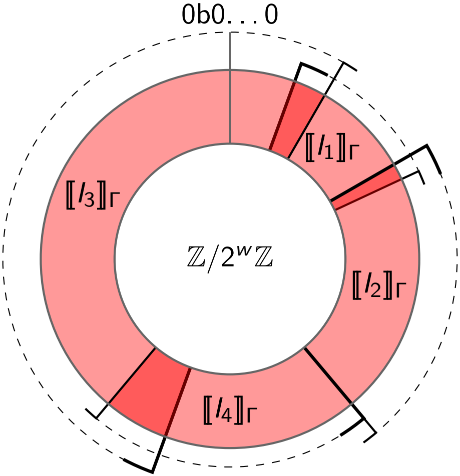

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Producing the forbidden intervals: preprocessing

- Step 1: only look at $\leq^u$, expressing $\leq^s$ and $\simeq$ in terms of $\leq^u$:

$$\begin{array}{lcl} t_1 \leq^s t_2 & \rightsquigarrow & t_1 + 2^{w-1} \leq^u t_2 + 2^{w-1} \\ t_1 \simeq t_2 & \rightsquigarrow & t_1 - t_2 \leq^u 0 \end{array}$$

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Then in order to compute a forbidden interval I from a constraint $t_1 \leq^u t_2$ where y has positive coefficients, we took inspiration from [JW16], but working with intervals modulo 2^w.

Producing the forbidden intervals

Coefficients of y in C are in $\{0, 1\}$ leaves 4 cases:

Normalised atom a	Forbidden interval that a (resp. $\neg a$) specifies for y		
	I_a	$I_{\neg a}$	**Condition**
$t_1 + y \leq^u t_2 + y$	$[-t_2; -t_1[$	$[-t_1; -t_2[$	$t_1 \not\simeq t_2$
	$\emptyset$	$[0; 0[$	$t_1 \simeq t_2$
$t_1 \leq^u t_2 + y$	$[-t_2; t_1 - t_2[$	$[t_1 - t_2; -t_2[$	$t_1 \not\simeq 0$
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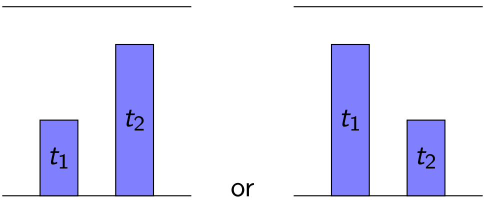
Illustrating the first line:



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Example 1: $\Gamma = \{x_1 \mapsto 0b0000\}$ and C_1 is literal $\neg(x_1 \leq^u y)$
$I_1 = [0; 0[$ (full interval $\mathbb{Z}/2^4\mathbb{Z}$), with condition $x_1 \simeq 0$
We take $\mathcal{C}$ to be the condition itself $x_1 \simeq 0$.
The lemma is $\neg(x_1 \leq^u y), x_1 \simeq 0 \models \bot$

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Example 2: $\Gamma = \{x_1 \mapsto 0b1100, x_2 \mapsto 0b1101, x_3 \mapsto 0b0000\}$ and

C_1 is	$\neg(y \simeq x_1)$	I_1 is	$[x_1; x_1 + 1[$	as $(0 \not\simeq -1)$
C_2 is	$(x_1 \leq^u x_3 + y)$	I_2 is	$[-x_3; x_1 - x_3[$	as $(x_1 \not\simeq 0)$
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Example 2, continued

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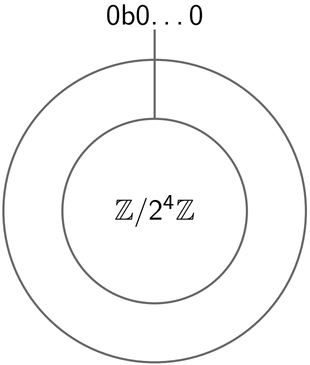

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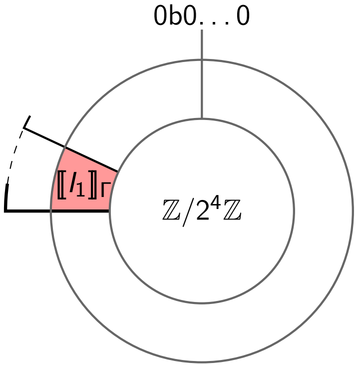

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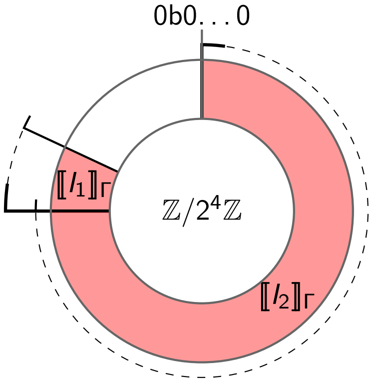

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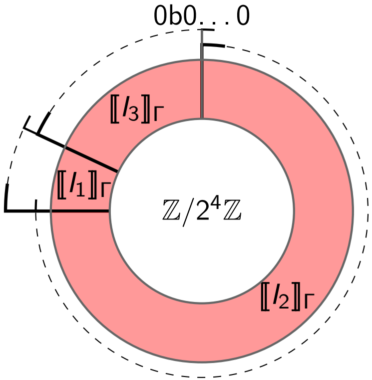

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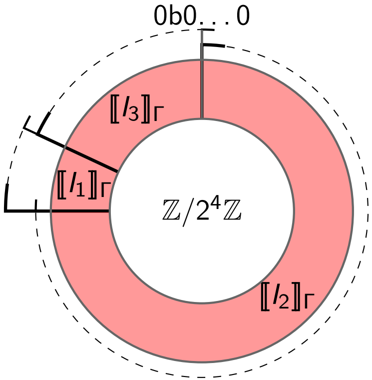

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Things to do in practice

- From the set $\{I_1, \ldots, I_m\}$ of intervals corresponding to constraints $C_1, \ldots, C_m$,
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  In Example 2, the formula $\mathcal{C}$,
  namely $(x_1{+}1) \in I_3 \ \wedge (-x_3) \in I_2 \wedge (x_1 - x_3) \in I_1$, becomes

  $$(x_1{+}1{-}x_2 <^u -x_3{-}x_2) \wedge (0 <^u x_1) \wedge (-x_3 <^u 1)$$

Algorithm for extracting a covering sequence

function $\text{SEQ\_EXTRACT}(\{I_1, \ldots, I_m\}, \Gamma)$
  output $\leftarrow$ () ▷ output initialised with the empty sequence of intervals
  longest $\leftarrow$ $\text{LONGEST}(\{I_1, \ldots, I_m\}, \Gamma)$ ▷ longest interval identified
  baseline $\leftarrow$ longest.upper ▷ where to extend the coverage from
  while $[\![\text{baseline}]\!]_\Gamma \notin [\![\text{longest}]\!]_\Gamma$ **do**
    $I \leftarrow \text{FURTHEST\_EXTEND}(\text{baseline}, \{I_1, \ldots, I_m\}, \Gamma)$
    output $\leftarrow$ output, I ▷ adding I to the output sequence
    baseline $\leftarrow$ I.upper ▷ updating the baseline for the next interval pick
  if $[\![\text{baseline}]\!]_\Gamma \in [\![\text{output.first}]\!]_\Gamma$ **then**
    return output ▷ the circle is closed without the help of longest
  return output, longest ▷ longest is used to close the circle

Also in the paper: generalisation with lower bits extraction

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  - because it can be used to express *0-extensions* or *sign-extensions*, which are very often used in the SMTLib library, e.g., to compare terms t_1 and t_2 with $\leq^u$, $<^u$, $\simeq$ when t_1 and t_2 have different bitwidths

Also in the paper: generalisation with lower bits extraction

- What is lower bits extraction?
  particular forms of extraction, $t[(m-1){:}0]$, denoted here $t\langle m\rangle$
  Mathematically, it is a projection of $\mathbb{Z}/2^w\mathbb{Z}$ to $\mathbb{Z}/2^m\mathbb{Z}$ ($m \leq w$)

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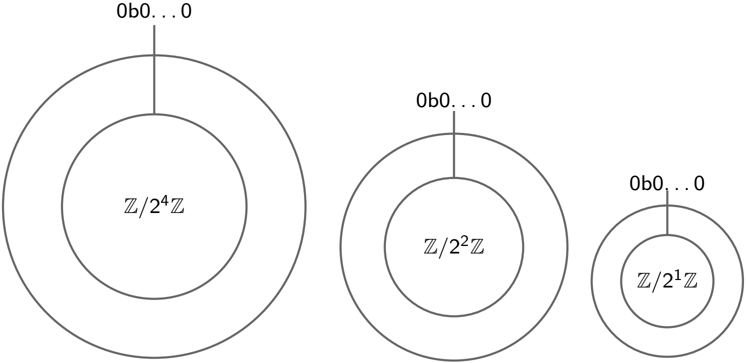
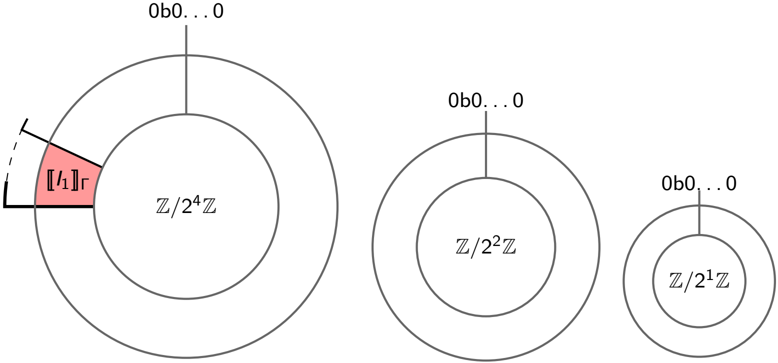
- Example? Variant of Example 2
  with model $\Gamma = \{x_1 \mapsto 0b1100, x_2 \mapsto 0b1101, x_3 \mapsto 0b0000\}$,
  and constraints

C_1	C_2	C_3	C_4
$\neg(y \simeq x_1)$	$(x_1 \leq^u x_3 + y)$	$(y\langle 2\rangle \leq^u x_2\langle 2\rangle)$	$(y\langle 1\rangle \simeq 0)$


0b0. . . 0
$\mathbb{Z}/2^4\mathbb{Z}$
0b0. . . 0
$\mathbb{Z}/2^2\mathbb{Z}$
0b0. . . 0
$\mathbb{Z}/2^1\mathbb{Z}$

Constraint C	C_1 $\neg(y \simeq x_1)$	C_2 $(x_1 \leq^u x_3 + y)$	C_3 $(y\langle 2\rangle \leq^u x_2\langle 2\rangle)$	C_4 $(y\langle 1\rangle \simeq 0)$
Forbidden interval I_C	$[x_1; x_1 + 1[$	$[-x_3; x_1 - x_3[$	$[x_2\langle 2\rangle + 1; 0[$	$[1; 0[$
Condition c	$(0 \not\simeq -1)$	$(x_1 \not\simeq 0)$	$(x_2\langle 2\rangle \not\simeq -1)$	$(0 \not\simeq -1)$
Concrete interval $[\![I_C]\!]_\Gamma$	$[0b1100; 0b1101[$	$[0b0000; 0b1100[$	$[0b10; 0b00[$	$[0b1; 0b0[$
Bit-width w_i	$w_1 = 4$		$w_2 = 2$	$w_3 = 1$
Forbidding values for	y		$y\langle 2\rangle$	$y\langle 1\rangle$



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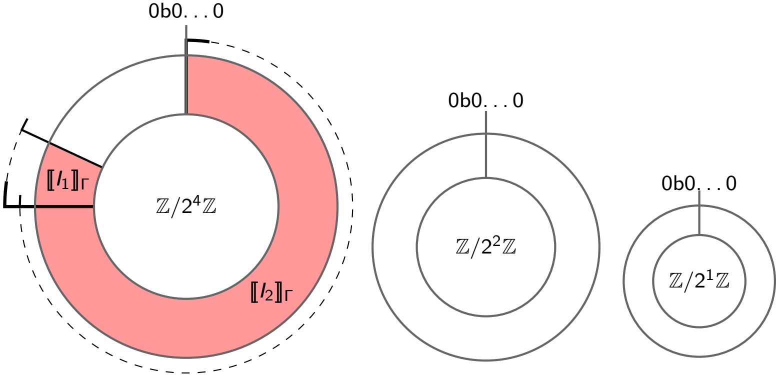

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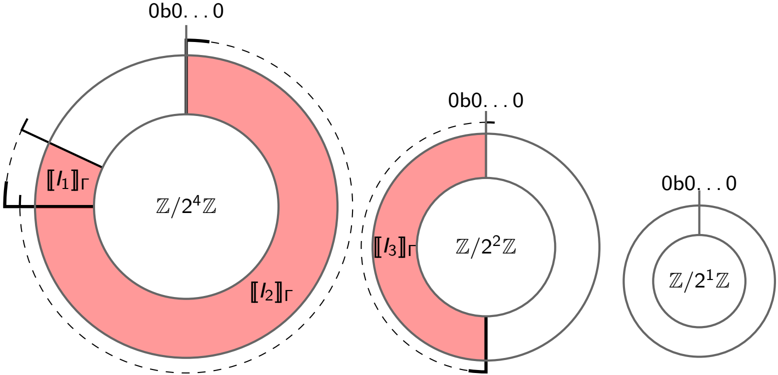

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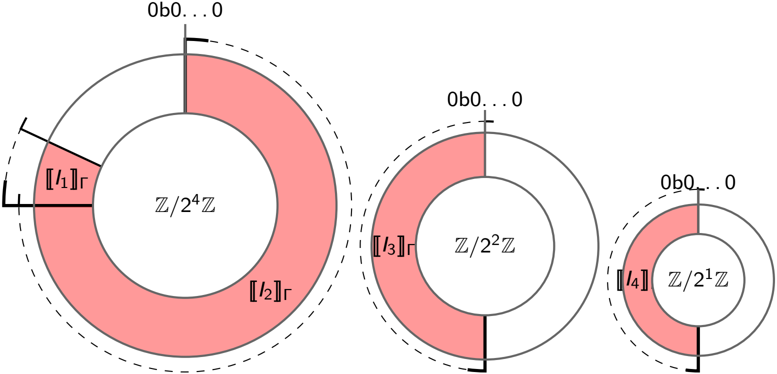

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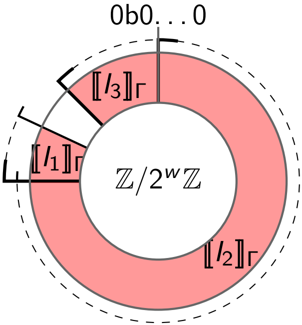
interpolation procedure handling intervals of multiple bitwidths;
we also have the optional part of an MCSAT theory, namely the propagation mechanism:



In case a single value is left uncovered, we produce term t and explanation $\mathcal{C}$ such that

$$C_1, \ldots, C_m, \mathcal{C} \models y \simeq t$$

which allows us to perform explainable propagations

Conclusion: experimentation

Still in progress, but performances are kind of predictable:
If the whole problem lies within this fragment of arithmetic...
...it performs very well!
If not, the first conflict that we bit-blast shoots us in the foot for the rest of the run.

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Some nice examples that we solve in less than a second (most are <0.2): the
`QF_BV/pspace/ndist.*.smt2` and the
`QF_BV/pspace/shift1add.*.smt2`
benchmarks fom the SMTLib library

Conclusion: where do we go from here?

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- Taking inspiration from other works on quantifier elimination in bitvector arithmetic [JC16].

Interpolation between two formulae

Here, specific form of interpolation, related to quantifier-elimination, serves MCSAT.
How can MCSAT, or our specific, model-driven form of interpolation can help solving the more standard form of interpolation between two formulae, remains to clarify, in connection with e.g., [Gri11].

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