Constrained Optimization Benchmark for Optimization Modulo Theories: a Cloud Resource Management Problem

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Outline

Motivation

Problem

Solution
- Solution Approaches
- Variables Encoding
- OMT Tools

Experimental Results

Discussion and Future Work

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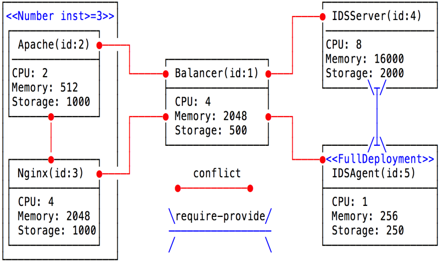

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- two Web Containers (e.g. Apache Tomcat or Nginx)
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- an IDSServer (Intrusion Detection System)
- an IDS Agent

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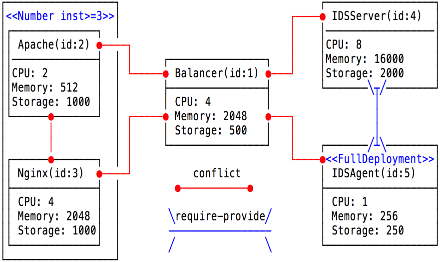

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- Conflicts: Balancer, Apache and Nginx cannot be deployed on the same VM
- Conflicts: Balancer and IDSServer needs exclusive use of machines
- Equal bound: exactly one Balancer has to be instantiated
- Lower bound: at least 3 instances of Apache and/or Nginx are required
- Require-provides: one IDSServer for 10 IDS Agents
- Full deployment: one instance of the IDS Agent on all VMs except for those containing the IDSServer and the Balancer
- Hardware constraints: components hardware requirements

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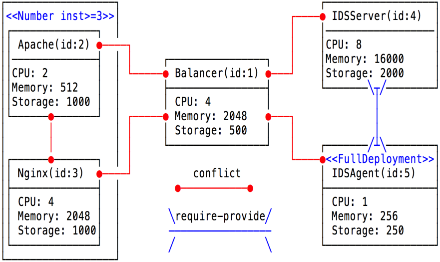

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Aim: find a set of virtual machines (VMs) which satisfy the components' requirements and lead to the minimum cost.

Cloud provider offers

Spot Instance Prices

Spot Instances | Defined Duration for Linux | Defined Duration for Windows

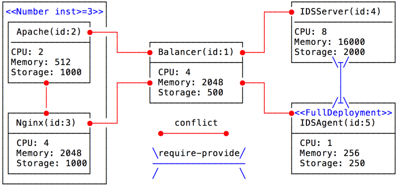
Region: EU (Ireland)

	Linux/UNIX Usage	Windows Usage
General Purpose - Current Generation		
t2.micro	$0.0038 per Hour	$0.0084 per Hour
t2.small	$0.0075 per Hour	$0.0165 per Hour
t2.medium	$0.015 per Hour	$0.033 per Hour
t2.large	$0.0302 per Hour	$0.0582 per Hour
t2.xlarge	$0.0605 per Hour	$0.1015 per Hour
t2.2xlarge	$0.121 per Hour	$0.183 per Hour
m3.medium	$0.0073 per Hour	$0.0633 per Hour
m3.large	$0.0306 per Hour	$0.1226 per Hour
m3.xlarge	$0.0612 per Hour	$0.2452 per Hour

Model	vCPU	CPU Credits / hour	Mem (GiB)	Storage
t2.nano	1	3	0.5	EBS-Only
t2.micro	1	6	1	EBS-Only
t2.small	1	12	2	EBS-Only
t2.medium	2	24	4	EBS-Only
t2.large	2	36	8	EBS-Only
t2.xlarge	4	54	16	EBS-Only
t2.2xlarge	8	81	32	EBS-Only

Remark: [snapshot from https://aws.amazon.com/ec2/] tens of thousands of price offers corresponding to different configurations and zones

Secure Web Container Use Case

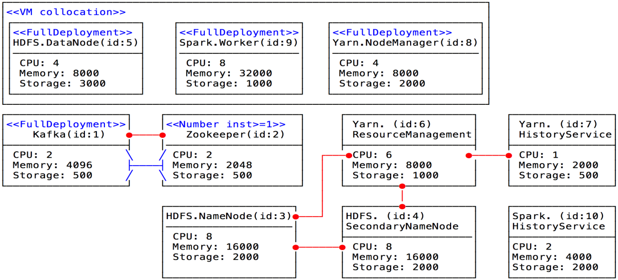


Example of a solution

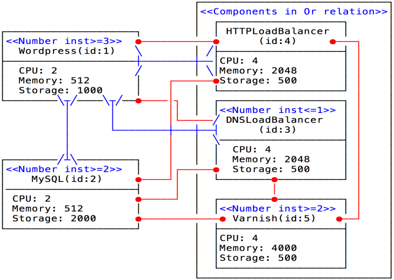
- VM_1 (CPU:8, RAM: 15 GB, Storage: 2000 GB, Price: 0.0526 $/hour): Nginx + IDS Agent
- VM_2 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.0283 $/hour): Balancer
- VM_3 (CPU:4, RAM: 30 GB, Storage: 2000 GB, Price: 0.0644 $/hour): IDSServer
- VM_4 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.0283 $/hour): Apache + IDS Agent
- VM_5 (CPU:4, RAM: 7.5 GB, Storage: 2000 GB, Price: 0.0283 $/hour): Apache + IDS Agent

Other use-cases considered

Oryx2 application



Wordpress application



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Input

- A set of N components ($\{C_1, ..., C_N\}$) satisfying:

- A set of M VMs ($\{V_1, ..., V_M\}$)

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- A mapping a of components to VMs

$$a_{ik} = \begin{cases} 1 & \text{if } C_i \text{ is assigned to } V_k \\ 0 & \text{if } C_i \text{ is not assigned to } V_k \end{cases}$$

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- which:
  - Satisfies the constraints induced by the interactions between components (structural constraints)
  - Satisfies the hardware requirements of all components (hardware constraints)
  - Minimizes the purchasing price

Problem Formalization (cont'd)

Default Constraints: $\sum_{i=1}^{N} a_{ik} \geq 1 \quad k = \overline{1, M}$

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Hardware Constraints

- H^{res} – required amount of resource (CPU/Memory/Storage)
- $H^{res_{prov}}$ – the corresponding characteristic of a VM included in an existing Cloud Provider offer.

$$\sum_{i=1}^{N} a_{ik} H_i^{res} \leq H_k^{res_{prov}}, \quad k = \overline{1, M}$$

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Structural Constraints

- Conflicts: two or more components cannot be deployed on the same VM

lin : $a_{ik} + a_{jk} \leq 1, k = \overline{1, M}, \forall (i, j) \text{s.t.} R_{ij} = 1$

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- Co-location: two or more components should be deployed on the same VM

$$a_{ik} = a_{jk}, \quad k = \overline{1, M}, \quad \forall (i, j) \text{ s.t. } D_{ij} = 1$$

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- Exclusive deployment: when from a set of q components only one should be deployed in a deployment plan

$$H(\sum_{k=1}^{M} a_{i_1 k}) + H(\sum_{k=1}^{M} a_{i_2 k}) + \ldots + H(\sum_{k=1}^{M} a_{i_q k}) = 1$$

$$H(u) = \begin{cases} 0 \text{ if } u = 0 \\ 1 \text{ if } u > 0 \end{cases}$$

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Structural Constraints (cont'd)

- Require-provides 1: C_i requires (consumes) at least n_{ij} instances of C_j and C_j can serve (provides) at most m_{ij} instances of C_i.

$$n_{ij} \sum_{k=1}^{M} a_{ik} \leq m_{ij} \sum_{k=1}^{M} a_{jk}, \quad n_{ij}, m_{ij} \in \mathbb{N}.$$

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- Require-provides 2: for each set of n instances of component C_j a new instance of C_i should be deployed

$$0 < n \sum_{k=1}^{M} a_{ik} - \sum_{k=1}^{M} a_{jk} \leq n, \quad n \in \mathbb{N}$$

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$$\sum_{k=1}^{M} (a_{ik} + H(\sum_{j, R_{ij}=1} a_{jk})) = M$$

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- Deployment with bounded number of instances: the number of instances corresponding to a set of deployed components, $\overline{C}$, should be equal, greater or less than some values

$$\sum_{i \in \overline{C}} \sum_{k=1}^{M} a_{ik} \ \langle \mathrm{op} \rangle \ n,$$

$$\langle \mathrm{op} \rangle \in \{=, \leq, \geq\}, \ n \in \mathbb{N}$$

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- $$\sum_{i=1}^{N} a_{ik} \geq 1 \wedge vmType_k = h \implies VP_k = Price^{Offer_h} \wedge vm_k^{HR_1} = HR_1^{Offer_h} \wedge \ldots \wedge vm_k^{HR_L} = HR_L^{Offer_h}$$

Example: the first offer is e.g. $h = 1$ and $(Price^{Offer_1}, HR_1^{Offer_1}, HR_2^{Offer_1}, HR_3^{Offer_1})$ is (9.152$, 64, 488MB, 8GB)

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- $$\sum_{i=1}^{N} a_{ik} = 0 \implies VP_k = 0, \quad k = \overline{1, M}$$

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  - ... the smallest price is not necessarily obtained by using the smallest number of bins

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[1] F. Micota, M. Erașcu and D. Zaharie, "Constraint Satisfaction Approaches in Cloud Resource Selection for Component Based Applications," 2018 IEEE 14th International Conference on Intelligent Computer Communication and Processing (ICCP), Cluj-Napoca, Romania, 2018, pp. 443-450.

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[2] M. Erașcu, F. Micota, and D. Zaharie. Influence of Variables Encoding and Symmetry Breaking on the Performance of Optimization Modulo Theories Tools Applied to Cloud Resource Selection. In G. Barthe, K. Korovin, S. Schulz, M. Suda, G. Sutcliffe, and M. Veanes, editors, LPAR-22 Workshop and Short Paper Proceedings, volume 9 of Kalpa Publications in Computing, pages 1–14. EasyChair, 2018.

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   - Satisfiability/Optimization Modulo Theory (SMT/OMT) + symmetry breaking
2. Approximate methods
   - Population-based metaheuristic[1]
     - Evolutionary algorithm that uses only mutation operator
     - Advantages: always provides a (sub)optimal solution
     - Drawback: low success rate in case of larger instances

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- testing of two state-of-the-art OMT solvers on different variable encodings and increasing problem dimension.

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  - the number of components instances in constraints like *deployment with bounded number of instances* and *require-provides* to be deployed;
  - the number of hardware/software constraints; we have used CPU, memory and storage.
- usage of different variable encodings

Variables Encoding

Variable name	Type				
VP, HR_t, vm^{HR_t}, $vmType$	`Real`	`Int`	`BV`	`Real`	~~`Int`~~
a	`Real`	`Int`	`BV`	`Bool`	~~`Bool`~~

- All variables have type `Real` (`RealReal`).
  - Add explicitly the constraint $a_{ik} = 0 \vee a_{ik} = 1$

  where $i = \overline{1, N}$, $k = \overline{1, M}$

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- All variables have type `Int`.
  - *Case* 1: $a_{ik} = 0 \vee a_{ik} = 1$ (`IntIntOr`)
  - *Case* 2: $0 \leq a_{ik} \leq 1$ (`IntIntLessThan`)

  where $i = \overline{1, N}$, $k = \overline{1, M}$

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- All variables are bitvectors (BV).
  - bitvectors of size 32 (the constants involved are big integer numbers)
  - unsigned versions of the relational operators since the terms are only positive

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  - *Case* 1: Encode the hardware constraints using the `ite` operator to make a_{ik} and HR_t^i having compatible types (`RealBool`).

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  - *Case* 2: Encode some of the constraints as cardinality (e.g. default contraints) and pseudo-boolean constraints (e.g. hardware constraints)

$$\text{default} : \sum_{i=1}^{N} a_{ik} \geq 1 \quad k = \overline{1, M}$$

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  - *Case* 1: Encode the hardware constraints using the ite operator to make a_{ik} and HR_t^i having compatible types (RealBool).

$$\sum_{i=1}^{N} a_{ik} \cdot HR_t^i \leq vm_k^{HR_t}, \quad vm_k^{HR_t}, HR_t^i \in \mathbb{R}_+, \quad k = \overline{1, M}, \quad t = \overline{1, L}$$

  - *Case* 2: Encode some of the constraints as cardinality (e.g. default contraints) and pseudo-boolean constraints (e.g. hardware constraints)

    Remark 1: Not of all the constraints involved are cardinality/pseudo-boolean constraints so the problem is not an MaxSMT problem, hence a solver for QF_LRA/QF_NRA must be used.

$$\bigvee_{h=1}^{ON} vmType_k = h$$

$$\sum_{i=1}^{N} a_{ik} \geq 1 \wedge vmType_k = h \implies VP_k = Price^{Offer_h} \wedge vm_k^{HR_1} = HR_1^{Offer_h} \wedge \ldots \wedge vm_k^{HR_L} = HR_L^{Offer_h}$$

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Remark 2:

- OptiMathSAT does not have support for the `RealPBC` encoding using the non-standard constructs like `atmost`, `atleast`, `exactly`, however they can be translated into `assert-soft` constraints (WiP).
- OptiMathSAT does not support non-linear constraints.
- In νZ, the cardinality constraints can be directly encoded, however the pseudo-boolean constraints can not (since a_{ik} is `Bool` and HR_t^i is `Real`).

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Solution for νZ:

- we used the ternary operator `ite`, transforming the type of a from `Bool` to `Real` in order to have compatible types for the variables involved in the constraints (`RealPBC`);
- we used the equivalent transformation of the hardware constraints (`RealPBCMultiObjectives`):

$$\sum_{i=1}^{N} a_{ik} \cdot HR_t^i \leq vm_k^{HR_t}, \quad vm_k^{HR_t}, HR_t^i \in \mathbb{R}_+, \quad k = \overline{1, M}, \quad t = \overline{1, L}$$

$$\rightsquigarrow$$

$$\neg a_{ik} \Rightarrow (a_{ik} H_i^{res} = 0) \ \wedge \ a_{ik} \Rightarrow (a_{ik} H_i^{res} = H_i^{res}) \wedge \texttt{minimize} \sum_{i=1}^{N} a_{ik} H_i^{res},$$

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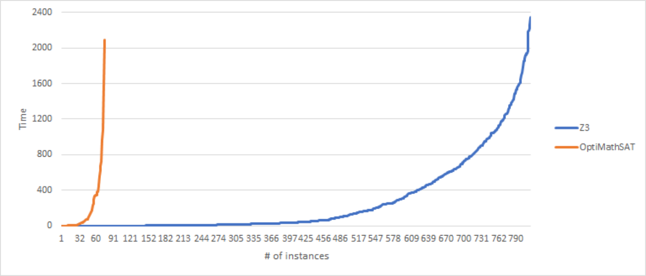

Figure: Comparison between νZ and OptiMathSAT

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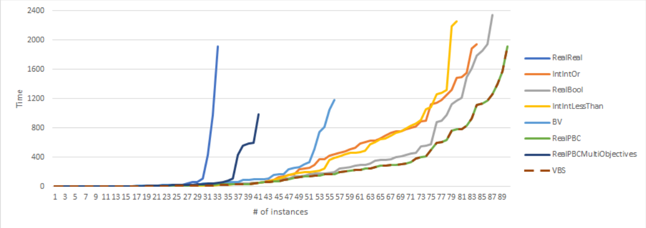

Figure: Scalability of νZ for different linear encodings

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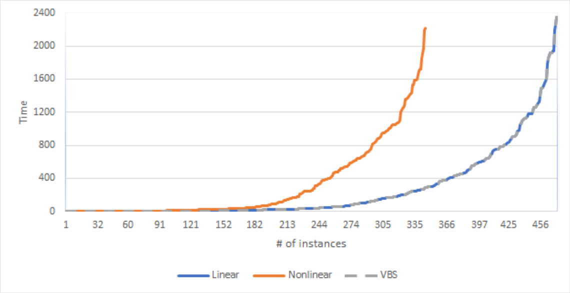

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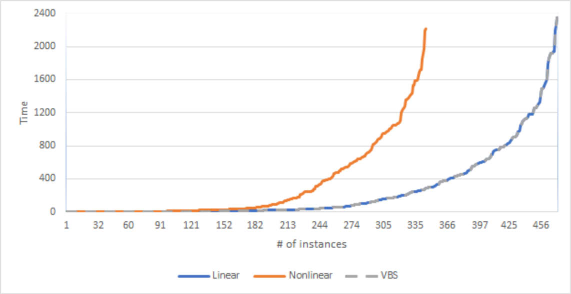

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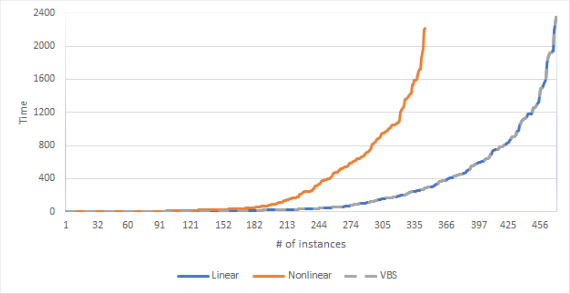

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- Benchmarks SMT-LIB-like at: `https://github.com/merascu/Optimization-Modulo-Theory/`

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Run 3	62.33	45.38	280.34	2484.58
Run 4	58.71	81.65	125.51	1790.21
Run 5	44.31	131.43	105.61	1955.71
Average	54.95	101.47	137.96	1916.48

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- It seems that for problems not involving massive arithmetic, the `RealPBC` encoding gives the best results.

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